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The Theory of Rest-Mass Quantisation

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Abstract

The general relativity concept of density-dependent space-curvature and the mass-energy relation of special relativity indicate a rest-mass quantisation rule which makes it possible to account for the interconversion of mass and energy in a simple manner and obtain the known quantum postulates as corollaries, thereby throwing new light on the nature of matter and radiation, the uncertainty principle, and the structure of elementary particles.

1. *Introduction*

In a recent paper (Chacko, 1974) I reported some simple relations between the masses of the known elementary particles, obtained on the basis of a rest-mass quantisation rule, and suggested a general equation which indicates the possibility for the existence of hitherto unidentified particles. In this paper I will attempt to find a basis for the rest-mass quantisation rule in the density-dependent space-curvature of general relativity and the mass-energy relation of special relativity and to obtain the other quantum postulates and the uncertainty principle as corollaries. The structure of elementary particles and that of space-time in general will be dealt with in subsequent papers.

The solution of the relativistic wave equation for the motion of an electron yields ± speed of light as eigenvalues for the instantaneous velocity of the electron (Dirac, 1958). This result, which is inevitable for all particles with mass, whether charged or neutral, has never been satisfactorily explained (Feynman, 1962). By choosing a general relativity model for the particle, I propose to show that this intrinsic motion with the speed of light can account for the rest-energy of the particle if its rest-mass is quantised so that the rest-energy multiplied by the period of motion with the speed of light in a closed cyclical path is a constant and that this quantisation also accounts for the inherent uncertainty in physical measurements, which makes it impossible to detect this intrinsic motion with the speed of light, if, for the basic particles, of which all other elementary particles are combinations, the constant is half the Planck's constant of action.

2. *A Model from General Relativity*

Using relativistic units of mass, length and time (Tolman, 1934a), we take as a model for particle, a space-time four-volume

$$S = \tfrac{8}{3}\pi^2 R^3 R_t \tag{2.1}$$

which may be considered as the four-surface of a five-sphere if $R = R_t$, as in the de Sitter model, or the four-volume of a hyper-torus if $R_t > R$; R being the radius of spatial curvature and $2\pi R_t$ the time taken to travel with the speed of light ($c = 1$) in a closed circular path of radius R_t. Assuming spatial spherical symmetry consider the model to be filled with a 'perfect fluid' of constant density ρ_{00}. Schwarzshild's interior solution (Tolman, 1934b) gives

$$\rho_{00} = \frac{3}{8\pi R^2} \tag{2.2}$$

Multiplying the space-time four-volume by this density, the action content of the model is obtained as

$$A = S\rho_{00} = \pi R R_t \tag{2.3}$$

Taking the volume of the spatial section as

$$V = \tfrac{4}{3}\pi R^3 \tag{2.4}$$

the rest-mass of the model is obtained as

$$m_0 = V\rho_{00} = \tfrac{1}{2}R \tag{2.5}$$

Substituting for R from (2.3),

$$m_0 = \frac{A}{2\pi R_t} \tag{2.6}$$

Converting to c.g.s. units

$$m_0 c^2 = \frac{A}{2\pi R_t/c}$$

or

$$m_0 c^2 (R_t/c) = \frac{A}{2\pi} \tag{2.7}$$

where R_t now has dimensions of length, so that $2\pi R_t/c$ gives the time for travelling along a closed circular path of radius R_t with the speed of light c, and may be considered as a period associated with the particle.

It is to be noted that the model is chosen to represent mass only. Properties like spin, magnetic moment, mean life etc., will not be considered in the present treatment.

3. *Rest-Energy and Rest-Mass Quantization*

We assign an intrinsic periodic motion in a circular path to the model particle when it is at rest in the classical sense—i.e., its experimentally determinable velocity, which is the average rate of change in the mean position in the laboratory frame of reference, taken over a period of time extending over several periods of the intrinsic motion, is zero—and show that the energy associated with this intrinsic periodic motion can account for the rest-energy of the particle if the instantaneous velocity of the periodic motion is of constant magnitude equal to the speed of light and the rest-mass of the particle is quantised so that the rest-energy multiplied by the period of motion is a constant. Since the particle is at rest in the classical sense we call the kinetic and potential energies associated with the periodic motion as 'rest-kinetic energy' and 'rest-potential energy' respectively and equate the sum of these energies to the rest-energy of the particle given by the mass-energy relation of special relativity.

Let u be the constant magnitude of the instantaneous velocity of the periodic motion in a circular path of radius r.

In equation (2.7), $u = c$ and $r = R_t$. So the rest-kinetic energy of the model particle is given by

$$T = \tfrac{1}{2} m_0 c^2 \tag{3.1}$$

and the rest-potential energy by

$$V = \int_0^{R_t} m_0 \frac{u^2}{r} \, dr \tag{3.2}$$

Equating the sum $E = T + V$ to the rest-energy of the particle we get

$$\tfrac{1}{2} m_0 c^2 + \int_0^{R_t} m_0 \frac{u^2}{r} \, dr = m_0 c^2 \tag{3.3}$$

It is to be emphasised here that u is the velocity of an intrinsic periodic motion associated with the model particle which is at rest in the classical sense, its mean position remaining fixed and measurable velocity zero, and that the mass does not change with u. Even when u has the magnitude of the velocity of light, the mass remains the same as the rest mass m_0 since the measurable velocity of the particle is zero ($v = 0$). Hence the validity of equations (3.1) and (3.2).

So from (3.3) we get that

$$\int_0^{R_t} m_0 \frac{u^2}{r} \, dr = \tfrac{1}{2} m_0 c^2 \tag{3.4}$$

This is possible only if

$$\frac{u}{r} = \frac{c}{R_t} = K \tag{3.5}$$

where K is a constant for the particle, so that

$$\int_0^{R_t} m_0 \frac{u^2}{r} dr = \int_0^{R_t} m_0 K^2 r \, dr = \tfrac{1}{2} m_0 K^2 R_t^2 = \tfrac{1}{2} m_0 c^2$$

We get from (2.7) and (3.5) that

$$K = \frac{c}{R_t} = \frac{m_0 c^2}{A/2\pi} \tag{3.6}$$

Since K is a constant for the particle, A and R_t must be constants.

Thus it is possible to account for the rest-energy of the particle in terms of the intrinsic periodic motion if, and only if, A is a constant and $u = c$. Since R_t is the radius of the circular path corresponding to the speed c in (2.7), it is a parameter, characteristic of the particle and $c/2\pi R_t$ gives the frequency associated with the particle when it is at rest. From (2.7) we see that this frequency multiplied by the constant A gives the rest-energy of the particle. So to solve the problem of the masses of elementary particles the constants A and R_t are to be evaluated.

Experimental data indicate that A is an integral or half-integral multiple of the Planck's constant of action h or combinations of these multiples, the multiplication factors being the so-called quantum numbers. It is possible to obtain semi-phenomenological relations between the masses of elementary particles in terms of these quantum numbers, based on the known interactions of these particles. Some of these results have been reported in Chacko (1974). A full theoretical treatment may have to await further developments from experimental investigations, though the already known symmetry laws and invariance principles may also guide in this direction.

The value of R_t in the case of elementary particles is about $1{\cdot}2 \times 10^{-13}$ cm, as reported in Chacko (1974), and this is nearly α^2 times smaller than the Bohr radius of the hydrogen atom, α being the fine structure constant. In equation (2.7) a circular path is assumed for simplicity. Paths may be circular, elliptical, or some other closed figure for particles at rest in the classical sense.

For the basic model we get, by giving A the value $\frac{1}{2}h$ in equation (2.7),

$$m_0 c^2 (R_t/c) = \frac{h}{4\pi} \tag{3.7}$$

The elementary particles can then be considered as combinations of these basic particles.

4. *Particles in Motion*

Particles in motion in the classical sense ($v \neq 0$) follow helical paths, the component of the instantaneous velocity c parallel to the axis of the helix giving the velocity of motion v.

Consider a particle of rest mass m_0 moving in the classical sense with a velocity v. If one turn of the helical path has a length λ, equation (2.7) becomes

$$m_0 c^2 (\lambda/c) = A \tag{4.1}$$

Or

$$\lambda = \frac{A}{m_0 c} \tag{4.2}$$

For a circular helix of radius r

$$\lambda = \left\{ (2\pi r)^2 + \left(\frac{v}{c}\right)^2 \lambda^2 \right\}^{1/2}$$

giving

$$\lambda = \frac{2\pi r}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}} \tag{4.3}$$

Combining (4.2) and (4.3) we get the inertial mass of the particle moving with velocity v as

$$m = \frac{m_0}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}} = \frac{A}{2\pi r c} \tag{4.4}$$

From the above equation Bohr's quantum condition for the electronic orbits can be obtained by putting

$$A = nh \qquad \text{and} \qquad r\frac{c}{v} = a \tag{4.5}$$

giving

$$mav = \frac{nh}{2\pi} \tag{4.6}$$

where a is the radius of the Bohr orbit.

The electrons travel with the speed of light in helical paths which have the shape of a coiled coil, the Bohr orbits representing the axes of the helical paths. Bohr orbits therefore give only the mean position of the electron at any given time, the velocity v being the average velocity of the electron around the nucleus. The instantaneous velocity is always c.

This picture of the electronic energy levels of the atom restores the vividness which has been absent from models based on the current interpretation of quantum mechanics, and shows how the electrons move with a velocity of constant magnitude c as required by the solution of the relativistic wave equation and at the same time conform to the statistical predictions of the theory. The details will be discussed in a subsequent paper.

5. *The Principle of Indeterminacy*

The inherent uncertainty in physical measurements is evident from (3.7). The smallest probe available for making physical measurements is the action unit obtained by putting $A = h/2$ in (2.7). So any measurement will cause a disturbance which cannot be less than this action unit which is given by

$$m_0 c^2 \times 2\pi R_t / c = \tfrac{1}{2} h \qquad (5.1)$$

We have no means of knowing whether this is added to or removed from the system by the process of measurement. So the uncertainty is at least double the smallest action unit. Hence the uncertainty principle is given by

$$\Delta E \, \Delta t \geqq \hbar \qquad (5.2)$$

where

$$\Delta E = m_0 c^2, \qquad \Delta t = R_t / c \qquad \text{and} \qquad \hbar = \frac{h}{2\pi}$$

This again explains why the periodic motion with the velocity of light in the case of material particles remains concealed. If the basic components of all particles conform to equation (5.1) so that

$$m_0 c^2 (R_t / c) \ngtr \frac{h}{4\pi} \qquad (5.3)$$

it will be impossible to detect this motion. Only average velocities for several such periods of time can be measured by experimental techniques.

6. *Inter-conversion of Matter and Radiation*

Let a particle with rest mass m_0 moving in the classical sense with a velocity v combine with its anti-particle which also may be assumed to be moving with the same velocity v.

Both obey equation (4.4) and we take the simplest case when $A = \tfrac{1}{2}h$, so that

$$m = \frac{\hbar}{2rc} \qquad (6.1)$$

The total energy of the two particles is given by

$$E = 2mc^2 \tag{6.2}$$

We will assume that there is no difference between a particle and its anti-particle except that their periodic circular motions are in the opposite sense, i.e. one is clockwise and the other anti-clockwise. These two circular motions combine to give a simple harmonic motion so that a photon is emitted with velocity c.

The amplitude of the S.H.M. is given by (6.1) as

$$a = r = \frac{\hbar}{2mc} \tag{6.3}$$

The frequency is given by

$$\nu = \frac{1}{T} = \frac{c}{2\pi r} = \frac{c}{2\pi} \times \frac{2mc}{\hbar} = \frac{2mc^2}{h} \tag{6.4}$$

We now calculate the energy of the photon on the assumption that it consists of two particles executing simple harmonic motion in the transverse direction while travelling forward with velocity c as in the classical theory. The total energy of the photon is the sum of the kinetic energy of propagation and the energy of S.H.M.

$$\text{Kinetic energy of propagation for the two particles, each of mass } m = 2 \times \tfrac{1}{2}mc^2 = mc^2 \tag{6.5}$$

Here again it is to be noted that the mass m changes only with velocity v, as given in (4.4), and this change affects only the amplitude a in (6.3) and the frequency in (6.4). The mass actually does not change when the particle and anti-particle combine to form the photon state because the magnitude of the velocity and hence the kinetic energy for the motion in the helical path in the particle state is the same as the velocity and kinetic energy for propagation in the photon state. Inertia vanishes because the circular motion changes to an S.H.M.

$$\text{Average kinetic energy of S.H.M. for the two particles} = \frac{(2m)\omega^2 a^2}{4} = \tfrac{1}{2}mc^2 \tag{6.6}$$

since $\omega a = \omega r = c$.

$$\begin{aligned}\text{Average potential energy of S.H.M. for the two particles} &= \pi^2(2m)\nu^2 a^2 \\ &= \pi^2(2m)\left(\frac{2mc^2}{h}\right)^2\left(\frac{\hbar}{2mc}\right)^2 \\ &= \tfrac{1}{2}mc^2 \end{aligned} \tag{6.7}$$

$$\text{The total energy of S.H.M.} = \tfrac{1}{2}mc^2 + \tfrac{1}{2}mc^2 = mc^2 \tag{6.8}$$

$$\text{Adding (6.5) and (6.8), total energy of the photon} = mc^2 + mc^2 = 2mc^2 \qquad (6.9)$$

Thus the energy of the photon emitted is the same as the combined energies of the particle and anti-particle given by (6.2).

From (6.2), (6.9) and (6.4) we get that for the photon, the energy is given by

$$E = h\nu \qquad (6.10)$$

which is the relation originally postulated by Planck.

Here we have taken only the simplest case. In pair production and annihilation involving actual elementary particles like electrons and positrons, other properties like spin have to be considered in order to account for phenomena like two-photon and three-photon annihilations.

7. *Conclusion*

The principle of rest-mass quantisation obtained by assuming an intrinsic periodic motion with the speed of light to account for the rest-energy of particles with non-zero rest-mass, leads to a clearer picture of wave-particle duality and the mechanism of the inter-conversion of matter and radiation. The motion with the speed of light manifests as inertia in material particles and the rest-mass is quantised so that this periodic motion with the speed of light remains hidden in the inherent uncertainty of physical measurements. Only the component of this velocity parallel to the axis of the helical path changes due to the action of a force and there is a corresponding change in the radius of the path and hence in the inertial mass of the particle so that the instantaneous velocity always remains equal to the speed of light.

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